COMPUTATIONAL ASPECTS OF LARGE-SCALE PROBLEMS IN DISCRETE MATHEMATICS

SEPTEMBER 20 – SEPTEMBER 24

2024

LIST OF ABSTRACTS

Program

Saturday morning, September 21

Generating and cataloging symmetric graphs

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Symmetry is a concept which plays a significant role in many areas of human activity. In mathematics, the desire to understand symmetry gave birth to modern group theory. Even today, groups are often studied in terms of their representations as symmetry groups of fixed mathematical objects, such as graphs. The study of groups acting on graphs, especially highly symmetric graphs, have resulted in many deep theories and ground-breaking results throughout mathematics. When studying a class of discrete objects, it is of profound importance to be able to construct complete lists of the representatives of this class up to a given size. On one hand, such lists (or censuses, as are often called) help us formulate and test conjectures. On the other hand, the lack of practical means of constructing such censuses indicates the lack of understanding of the theory. Thus, development of theory (and, of course, computational resources) enables constructions of complete censuses of objects, while censuses themselves facilitate and motivate further theoretical achievements. Attempts of constructing census of graphs with high level of symmetry began in early 1930s, when Foster started collecting examples of arc-transitive graphs of valence 3. His work, now known as Foster's census, has been a valuable source of information for graph and group theorists for many decades. Several legendary mathematicians have been involved in constructions of catalogues of graphs of specific symmetry types, such as William Tutte, Harold Coxeter, John Conway etc. In the last few decades, Foster's original work was successfully upgraded in several directions. The aim of the talk is to present the theory and methods behind constructions of these classical censuses, give a few practical demonstrations using modern computer algebra systems, and present some of the more recent achievements in this area.

On Siamese Color Graphs

Nina Hronkovičová[*], *Martin Mačaj*
[*]*DAG, Comenius University*

A Siamese color graph is an edge decomposition of a complete graph into strongly regular subgraphs sharing a spread. Using a computer aided exhaustive search we complete the classification of Siamese color graphs on 40 vertices.

A note on parabolic and linear one-factorizations of the complete graph K_{p+1}

György Kiss, Gábor Korchmáros, Federico Romaniello and Valentino Smaldore[*]
[*]*Università degli Studi di Padova*

One-factorizations of the complete graph K_n have wide applications, as an example they are often used for scheduling round-robin tournaments with n teams. In this note, we characterize parabolic and linear one-factorizations of complete graphs K_{p+1}, when p is an odd prime. This class of one-factorizations arises from the geometry of conics and lines in the affine plane $AG(2, p)$. We also include Magma computations for the cases $p \leq 19$.

Partial automorphisms and level of symmetry of graphs

Valter Cingel, Tatiana Jajcayová and Ján Pastorek[*]
[*]*DAI, Comenius Univesity*

In our presentation, we address several open questions concerning the symmetry levels of graphs posted by Cingel, Gál & Jajcayová (2023), and derive additional results using both computer aided and theoretical methods. We improve the best previously known lower bound for the symmetry levels of general graphs by proving that the symmetry level of any finite simple graph is at least $\frac{1}{2}$. In case of disconnected graphs without a unique isolated vertex, we prove that the symmetry level of such graphs is at least $\frac{3}{4}$. Furthermore, we present graphs that provide an answer to Question 3 posted by Cingel, Gál & Jajcayová by showing that higher symmetry level does not necessarily imply a larger number of partial automorphisms. We take the initial steps toward answering the main question of Cingel, Gál & Jajcayová. Finally, we discuss the relation between a measure of asymmetry introduced by Erdős & Rényi (1963) and the level of symmetry of graphs considered in our paper.

Integer sequences from k-iterated line digraphs

Dominika Závacká[*], *Cristina Dalfó and Miquel Angel Fiol*
[*]*DAI, Comenius University*

In this presentation, we focus on integer sequences corresponding to the number of vertices in k-iterated line digraphs. We begin by introducing the core concepts

related to digraphs. Then, we describe a method, proposed by Dalfó and Fiol, for calculating the order of k-iterated line digraphs. We explore various families of digraphs, such as De Bruijn, Kautz, Cyclic Kautz, and Square-free digraphs. To generate integer sequences representing the number of vertices in k-iterated line digraphs, we implement an algorithm that constructs induced subdigraphs by not allowing vertices containing forbidden subwords. The results include comparisons of the obtained integer sequences with those in the OEIS database and identification of new integer sequences. Our algorithm is implemented in the computational system GAP.

Saturday afternoon, September 21

Some old and new results and open problems in Finite Geometry

Gábor Korchmáros
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The earliest, mostly sporadic investigations in Finite geometry date back to one hundred years ago, at least. It was in the mid-nineteen-fifties that its systematic study was initiated by the pioneering work of the famous Italian mathematician Beniamino Segre. In the Preface of his monograph "Projective Geometries over Finite Fields", Clarendon Press, Oxford (1979), and Springer Verlag (1998), J.W.P. Hirschfeld pointed out that Segre's work "stimulated a considerable amount of research, which still leaves many easily stated, but apparently difficult problem unsolved." The guiding idea in Segre's work, and in several contemporary researches, is to figure out and then study those objects in a finite plane which can be defined by combinatorial abstraction of some well-known objects in the real or complex plane. Such objects, especially k-arcs, ovals, unitals, and k-nets, still play an important role in many investigations also in connection to other areas in Discrete Mathematics and its applications. This gives a motivation to present and discuss several results and open problems.

Vertex-girth-regular graphs

Robert Jajcay
DAG, Comenius University

A vertex-girth-regular $vgr(v, k, g, \lambda)$-graph is a k-regular graph Γ of girth g and order v in which every vertex belongs to exactly λ cycles of length g. While all vertex-transitive graphs are necessarily vertex-girth-regular, the majority of vertex-girth-regular graphs are not vertex-transitive. Similarly, while many of the smallest k-regular graphs of girth g, the so-called (k, g)-cages, are vertex-girth-regular, infinitely many vertex-girth-regular graphs of degree k and girth g exist for many (but not all) pairs k, g. Due to these connections, the study of vertex-girth-regular graphs promises insights into the relations between the classes of extremal, highly symmetric, and locally regular graphs of given degree and girth. Moreover, the class of vertex-girth-regular graphs includes two other previously studied classes introduced in this context, namely, girth-regular and edge-girth-regular graphs; both of which proved to be of interest to the scientific community.

In our presentation, we investigate the fundamental properties of $vgr(v, k, g, \lambda)$-graphs, specifically the relations necessarily satisfied by the parameters v, k, g and λ to admit the existence of a corresponding vertex-girth-regular graph, present constructions of infinite families of $vgr(v, k, g, \lambda)$-graphs, and establish lower bounds on the number v of vertices in a $vgr(v, k, g, \lambda)$-graph. The talk also includes computational results determining the orders of smallest cubic and quartic graphs of small girths.
The presentation is based on joint work with Jorik Jooken and István Porupsánszki.

Sunday, September 22

On the Moore-Penrose pseudo-inversion of block symmetric matrices and its application in graph theory

Daniel Ševčovič
DAMS, Comenius University

In this talk we analyze the Moore-Penrose pseudo-inversion of symmetric real matrices with application in the graph theory. We introduce a novel concept of positively and negatively pseudo-inverse matrices and graphs. We also give sufficient conditions on the elements of a block symmetric matrix yielding an explicit form of its Moore-Penrose pseudo-inversion. Using the explicit form of the pseudo-inverse matrix we can construct pseudo-inverse graphs for a class of graphs which are constructed from the original graph by adding pendent vertices or pendant paths. This is a joint work with Soňa Pavlíková.

Circular chromatic index of small snarks

Dušan Bernát[*] *and Ján Mazák*
[*]*DI, Comenius University*

This contribution verifies and extends previous computational results on circular chromatic index of small snarks, gives further support for certain conjectures, and discusses various practical aspects of using SAT solvers to solve graph colouring problems (e.g. the impact of solver choice and Boolean formula construction on running time).

Forbidden paths and cycles in the undirected underlying graph of a 2-quasi best match graph

Annachiara Korchmaros
Universität Leipzig

The undirected underlying graph of a 2-quasi best match graph (2-qBMG) is proven not to contain any induced graph isomorphic to P_6 or C_6. This new feature allows for the investigation of 2-BMGs further by exploiting the numerous known results on P_6 and C_6 free graphs together with the available polynomial algorithms developed for their studies. In this direction, there are also some new

contributions about dominating bicliques and certain vertex decompositions of the undirected underlying graph of a 2-qBMG.

Upper Bounds and Probability Heuristic Estimates of r-Regular Families of Permutations

Pavol Kollár[*] *and Martin Mačaj*
[*]*DAI, Comenius University*

A sure-fire way of answering "How many of these objects are there?" is to generate them all and simply keeping a counter. However, if it so happens that the true count is about 10^{24}, this way of doing it becomes a "sure-fire way of getting nowhere". This is exactly what happens with so called r-regular families of permutations, which are a generalisation of the notion of transitivity in groups. They also measure how Cayley-like a vertex-transitive graph is. Therefore, another approach is required for this (and many other) cases. In this article, we shall describe a general-purpose algorithm that utilises complex cyclotomic numbers to give upper bounds to the counts of these families. Additionally, we present a probabilistic algorithm to give heuristic estimates of the true counts of these families.